

Stability of the 3-form field during inflation

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We consider the minimally coupled 3-form field which has been considered as a candidate to realize inflation. We have studied the conditions to avoid ghosts and Laplacian instabilities and found that some classes of potentials, e.g. the Mexican-hat one, will in general be unstable. We propose other classes of potentials which are instead free from any instability, drive a long-enough slow-roll regime followed by an oscillatory epoch, and as a consequence, can provide successful inflation.

I. INTRODUCTION

The inflation paradigm was introduced in 1980 as a way to solve different issues, namely: the magnetic monopoles, the flatness, and the horizon problems [1]; however, it can also account for the observed temperature anisotropies in the Cosmic Microwave Background (CMB) [2] as well as the galaxy power spectrum [3]. In other words a sufficiently long stage of accelerated expansion has been proposed as a way to solve all these problems at the same time. In order to explain this period of accelerated expansion, some new physics is introduced, and a scalar field [1, 4] (or more than one [5]) is commonly used. However, the real mechanism for inflation is yet unknown, so it is interesting to explore different possibilities which in general may lead to different predictions for several inflationary observables (i.e. spectral index, tensor-to-scalar ratio [7, 8], non-gaussianity parameter [9–12]).

Since fundamental scalar fields have not been discovered yet in nature, the idea of inflation might well be realized by other, higher-form fields. For example, vector inflation (or one-form inflation) has been intensively investigated [13–16]. Unfortunately, most of the vector-field models encounter instabilities [17–20]. More in general, the N-form field inflation has been also investigated [21, 22] and one of the results is that one-form and two-form fields are not stable, whereas the three-form field can be stable [22]. Note also, that the four-form field models correspond to the $f(R)$ theories [22, 23].

Recently, a form of inflation based on the evolution of a 3-form has been studied [25–28]. The origin of such a non-standard form for the inflaton may come from a high-energy-scale theory, such as string-theory. Indeed it is interesting to study such a model, as it may provide an alternative way to obtain inflation. Since the essence of the 3-form is by construction different from a single scalar field, we expect this difference to play some role both at background and perturbation levels.

In fact, in this paper we will study the stability of a minimally coupled 3-form during inflation with a general expression for the potential. We will then find the conditions which avoid ghosts and Laplacian instabilities (i.e. we require a positive kinetic term and a non-negative speed of propagation for the independent linear perturbation modes). Once these conditions are obtained, we reconsider some models which have been recently introduced [21, 22], and show that, if the potential is not carefully chosen, both ghosts and Laplacian instabilities will occur.

Hence, we provide some classes of potentials, which, by construction, are instead free from these instabilities, and, in this context, we study their background evolution, in order to confirm that a slow-roll period of inflation is then followed by a regime where the 3-form oscillates, ending inflation. We will discuss the details of reheating, and the bounds on the inflationary parameters (spectral index, tensor-to-scalar ratio, and non-gaussianities) in a future work.

The paper is organized as follows. In section II, we introduce the Lagrangian of the model and write down the equations of motion. Linear perturbation theory for this model is studied in section III, where we give the no-ghost conditions and the squared speed of propagation for the scalar, vector and tensor modes. We present some classes of potentials which make the model free from ghosts and Laplacian instabilities in section IV, where we also show that a slow-roll period of inflation is followed by an oscillatory regime which ends inflation. We write our conclusions in section V.

II. THE MODEL AND THE BACKGROUND EQUATIONS OF MOTION

Let us start with the following action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{48} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} - V(A_{\alpha\beta\gamma} A^{\alpha\beta\gamma}) \right], \quad (1)$$

where $A_{\alpha\beta\gamma}$ is a 3-form, and $\mathbf{F} = d\mathbf{A}$ is its Maxwell tensor [24], whose components can be written as

$$F_{\mu\nu\rho\sigma} = \nabla_\mu A_{\nu\rho\sigma} - \nabla_\sigma A_{\mu\nu\rho} + \nabla_\rho A_{\sigma\mu\nu} - \nabla_\nu A_{\rho\sigma\mu}. \quad (2)$$

A. The background

Let us now consider a flat Friedmann-Lemaître-Robertson-Walker (FLRW) manifold whose metric element can be written as

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2, \quad (3)$$

and, on this background, considering Eq. (A1) in Appendix, the background 3-form $A_{\alpha\beta\gamma}$ can be written as

$$A_{0ij} = 0, \quad A_{ijk} = a^3 \epsilon_{ijk} X, \quad (4)$$

where ϵ_{ijk} is the three-dimensional Levi-Civita symbol (with $\epsilon_{123} = 1$). Let us define the following quantities

$$V = V(y), \quad V_{,y} \equiv \frac{dV(y)}{dy}, \quad V_{,yy} \equiv \frac{d^2V(y)}{dy^2}, \quad \text{where } y \equiv A_{\alpha\beta\gamma} A^{\alpha\beta\gamma}. \quad (5)$$

On a FLRW background, we have $y = 6X^2$. As a consequence of these definitions, on FLRW, we have

$$V(-X) = V(X), \quad (6)$$

$$V_{,y} = \frac{dX}{dy} V_{,X} = \frac{V_{,X}}{12X}, \quad (7)$$

$$\dot{V} = 12X \dot{X} V_{,y} \quad (8)$$

$$V_{,yy} = \frac{1}{12X} \frac{d}{dX} \left(\frac{V_{,X}}{12X} \right) = \frac{XV_{,XX} - V_{,X}}{144X^3}, \quad (9)$$

so that we will restrict the form of the potential V to even functions of X . In this case the Friedmann equation can be written as

$$E_1 \equiv 3M_{\text{Pl}}^2 H^2 - \rho_X = 0, \quad (10)$$

where

$$\rho_X = \frac{1}{2} \dot{X}^2 + V + \frac{9}{2} H^2 X^2 + 3HX\dot{X} = \frac{1}{2} Y^2 + V, \quad (11)$$

is the effective energy density of the 3-form, and we have defined $Y \equiv \dot{X} + 3HX$. The second Einstein equation reads as follows

$$E_2 \equiv M_{\text{Pl}}^2 (2\dot{H} + 3H^2) + p_X = 0, \quad (12)$$

where p_X is the 3-form effective pressure defined as

$$p_X = - \left(\frac{1}{2} \dot{X}^2 + V + 3HX\dot{X} + \frac{9}{2} H^2 X^2 - 12V_{,y} X^2 \right) = 2V_{,y} y - \rho_X. \quad (13)$$

The equation of motion for the field gives

$$E_X \equiv \ddot{X} + 3H\dot{X} + 3X\dot{H} + 12V_{,y}X = \dot{Y} + 12XV_{,y} = 0. \quad (14)$$

The equations of motion are not all independent, due to Bianchi identities: indeed we have

$$\dot{E}_1 + 3H(E_1 - E_2) + YE_X = 0. \quad (15)$$

One consequence of the equations of motion is

$$M_{\text{Pl}}^2 \dot{H} = -V_{,y} y, \quad (16)$$

so that the universe will be super-accelerating when $V_{,y} < 0$.

III. LINEAR PERTURBATION THEORY

A. Scalar modes

Let us consider now the metric for the scalar perturbations in the following form [29]

$$ds^2 = -(1 + 2\alpha)dt^2 + 2\partial_i\psi dt dx^i + a^2(1 + 2\Phi)d\mathbf{x}^2, \quad (17)$$

where we picked a spatial gauge so that the three-dimensional metric is diagonal. As for the 3-form, by using once more Eq. (A1) given in Appendix, we can use a time gauge to fix the scalar perturbations as [27]

$$A_{0ij} = a\epsilon_{ijk}\partial_k\beta(t, \mathbf{x}), \quad A_{ijk} = a^3\epsilon_{ijk}X(t). \quad (18)$$

By expanding the action at second order in the fields we obtain

$$\begin{aligned} S^{(2)} = \int dt d^3x a^3 \bigg\{ & \frac{6V_{,y}X^2}{a^2}(\partial\psi)^2 - 2M_{\text{Pl}}^2(H\alpha - \dot{\Phi})\frac{\partial^2\psi}{a^2} \\ & + \frac{1}{2}\frac{(\partial^2\beta)^2}{a^4} + 6V_{,y}\frac{(\partial\beta)^2}{a^2} + (Y\alpha + 12V_{,y}X\psi + 3Y\Phi)\frac{\partial^2\beta}{a^2} \\ & - \frac{1}{2}(6M_{\text{Pl}}^2H^2 - Y^2)\alpha^2 + \left[6M_{\text{Pl}}^2H\dot{\Phi} - 2M_{\text{Pl}}^2\frac{\partial^2\Phi}{a^2} + 3(Y^2 + 12V_{,y}X^2)\Phi\right]\alpha \\ & - 3M_{\text{Pl}}^2\dot{\Phi}^2 + M_{\text{Pl}}^2\frac{(\partial\Phi)^2}{a^2} + \frac{9}{2}(Y^2 - 12V_{,y}X^2 - 144V_{,yy}X^4)\Phi^2 \bigg\}. \end{aligned} \quad (19)$$

At a first look, this action has important differences with the general action (for the perturbations) of scalar tensor theories [30]. First of all the presence of the terms $(\partial\psi)^2$ and Φ^2 which, for a second-order general scalar-tensor theory, vanish after using the equations of motion. Both these terms now vanish only when the 3-form is absent. Furthermore the field β is not dynamical and it can be integrated out in Fourier space (together with α and ψ).

In order to remove these auxiliary fields it is convenient to work in Fourier space: in this case, we can integrate out the fields α , ψ , and β , by using their own equations of motion. In Fourier space, with $\Phi(t, \mathbf{x}) = (2\pi)^{-3/2} \int d^3k \tilde{\Phi}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}$, with reality condition $\tilde{\Phi}_{-\mathbf{k}} = \tilde{\Phi}_{\mathbf{k}}^*$, the equations of motion for the constraints give

$$12V_{,y}X^2\psi + 2M_{\text{Pl}}^2(H\alpha - \dot{\Phi}) - 12V_{,y}X\beta = 0, \quad (20)$$

$$(Y^2 - 6M_{\text{Pl}}^2H^2)\alpha + \frac{2M_{\text{Pl}}^2Hk^2\psi}{a^2} + 6M_{\text{Pl}}^2H\dot{\Phi} + 2M_{\text{Pl}}^2\frac{k^2\Phi}{a^2} + 3(Y^2 + 12V_{,y}X^2)\Phi - \frac{Yk^2\beta}{a^2} = 0, \quad (21)$$

and

$$\frac{k^2}{a^2}\beta + 12V_{,y}\beta - Y\alpha - 12V_{,y}X\psi - 3Y\Phi = 0, \quad (22)$$

where we omitted the tilde of the Fourier modes for simplicity. This last equation can be solved for β as

$$\beta = \frac{a^2(Y\alpha + 12V_{,y}X\psi + 3Y\Phi)}{k^2 + 12V_{,y}a^2}, \quad (23)$$

so that we also have

$$\psi = \left(\frac{M_{\text{Pl}}^2}{6V_{,y}X^2} + \frac{2a^2M_{\text{Pl}}^2}{X^2k^2} \right) \dot{\Phi} + \frac{3a^2Y}{Xk^2} \Phi - \left(\frac{M_{\text{Pl}}^2H}{6V_{,y}X^2} + \frac{a^2(2M_{\text{Pl}}^2H - XY)}{k^2X^2} \right) \alpha, \quad (24)$$

and finally

$$\begin{aligned} \alpha = & \frac{M_{\text{Pl}}^4Hk^2 + 6M_{\text{Pl}}^2V_{,y}a^2(3HX^2 + 2M_{\text{Pl}}^2H - XY)}{M_{\text{Pl}}^2H[M_{\text{Pl}}^2k^2H + 6V_{,y}a^2(3HX^2 + 2M_{\text{Pl}}^2H - 2XY)]} \dot{\Phi} \\ & + \frac{6V_{,y}M_{\text{Pl}}^2k^2X^2 + 18V_{,y}a^2X(6X^3V_{,y} + M_{\text{Pl}}^2HY)}{M_{\text{Pl}}^2H[M_{\text{Pl}}^2k^2H + 6V_{,y}a^2(3HX^2 + 2M_{\text{Pl}}^2H - 2XY)]} \Phi. \end{aligned} \quad (25)$$

By substituting these expressions into the action written in Fourier space, we find

$$\tilde{S} = \int dt d^3k Q(t, k^2) \left[\dot{\Phi}_{\mathbf{k}}\dot{\Phi}_{-\mathbf{k}} - c_X^2(t, k^2) \frac{k^2}{a^2} \Phi_{\mathbf{k}}\Phi_{-\mathbf{k}} \right]. \quad (26)$$

1. Ghost conditions

The no-ghost condition is then found to be

$$Q > 0, \quad (27)$$

where

$$Q = \frac{6a^5 M_{\text{Pl}}^2 V_{,y} Y^2}{M_{\text{Pl}}^2 k^2 H^2 + 6 V_{,y} a^2 (3 H^2 X^2 + 2 M_{\text{Pl}}^2 H^2 - 2 X Y H)}. \quad (28)$$

It should be noticed that condition (27) should hold at all times during inflation, whether or not the trajectory is in a slow-roll regime. On using the equations of motion (by replacing $M_{\text{Pl}}^2 H^2$ with the Friedmann equation and then $3 H X = Y - \dot{X}$) we find that

$$3 H^2 X^2 + 2 M_{\text{Pl}}^2 H^2 - 2 X Y H = \frac{1}{3} \dot{X}^2 + \frac{2}{3} V, \quad (29)$$

so that

$$Q = \frac{6a^5 M_{\text{Pl}}^2 V_{,y} Y^2}{M_{\text{Pl}}^2 k^2 H^2 + 2 V_{,y} a^2 (\dot{X}^2 + 2V)}. \quad (30)$$

This quantity must be positive for all k 's. For high k , we find the condition $V_{,y} > 0$. This condition must be satisfied along the trajectory of motion. In some cases, it may be possible that for some (positive) values of y , $V_{,y}$ is negative, but such values of y are never reached: in this case the model can still be viable. We note here that the condition $V_{,y} > 0$, on using Eq. (16), forbids the dynamics to be super-accelerating.

For low k 's we find another requirement, that is $\dot{X}^2 + 2V > 0$. Once more, this condition must be satisfied along the trajectory of motion. The bottom line is that the two conditions $V \geq 0$, $V_{,y} > 0$ are sufficient conditions for not having ghosts. If these conditions are not satisfied for all (positive) y 's, one should check that, at least for the values of y along the trajectory of motion for the model, the above mentioned conditions still hold.

2. Speed of propagation

The speed of propagation is found as the large- k limit of $c_X^2(t, k^2)$ of Eq. (26). One can show that the speed of propagation, on using the background equations of motion, is given as

$$c_X^2 = \lim_{k \rightarrow \infty} c_X^2(t, k^2) = 1 + \frac{2V_{,yy}y}{V_{,y}} = \frac{X V_{,XX}}{V_{,X}}. \quad (31)$$

The speed of propagation found here corresponds to the one found by Koivisto and Nunes [27]. In general, only the simple quadratic potential $V \propto y$, implies a propagation with speed of light for all dynamics. Since $y \geq 0$, then a sufficient condition to avoid also Laplacian instabilities (besides the ghosts, $V_{,y} > 0$) is $V_{,yy} \geq 0$.

B. Vector modes

Let us define the metric perturbation for the vector modes as

$$\delta g_{0i} = a G_i, \quad \text{and} \quad \delta g_{ij} = a^2 (C_{i,j} + C_{j,i}), \quad (32)$$

where $G_{i,i} = 0 = C_{i,i}$. We will also choose a gauge for which the 3-form has no vector perturbations (uniform field vector-gauge). This choice completely fixes the gauge degrees of freedom. In this case, one can show that the action for the vector modes becomes

$$S = \int dt d^3x \left[6a^5 V_{,y} X^2 \dot{C}_i \dot{C}_i + 12a^4 V_{,y} X^2 \dot{C}_i Z_i + \frac{1}{4} M_{\text{Pl}}^2 a (\partial_j Z_i)(\partial_j Z_i) + 6a^3 V_{,y} X^2 Z_i Z_i \right], \quad (33)$$

where we introduced the field $Z_i = G_i - a \dot{C}_i$. By introducing Fourier modes, it is possible to integrate out the field Z_i as

$$\tilde{Z}_i(t, \mathbf{k}) = -\frac{24a^3 V_{,y} X^2 \dot{C}_i(t, \mathbf{k})}{M_{\text{Pl}}^2 k^2 + 24a^2 V_{,y} X^2}, \quad (34)$$

so that the action for the vector modes becomes

$$S = \int dt d^3k Q_V(t, k^2) \left[\dot{C}_i(t, \mathbf{k}) \dot{C}_i(t, -\mathbf{k}) \right], \quad (35)$$

so that it is clear that the vector modes do not propagate.

1. Ghost condition

The ghost condition for the vector modes corresponds to $Q_V > 0$, that is

$$Q_V = \frac{6k^2 a^5 M_{\text{Pl}}^2 V_{,y} X^2}{M_{\text{Pl}}^2 k^2 + 24a^2 V_{,y} X^2} > 0, \quad (36)$$

implying

$$V_{,y} > 0, \quad (37)$$

which coincides to one of the conditions already found for the scalar modes.

C. Tensor modes

The tensor modes are not affected by the presence of the 3-form, as this latter one is minimally coupled to gravity and it does not possess tensor degrees of freedom. To show this more in detail, we choose the tensor perturbations as $\delta g_{ij} = h_{ij}^T = h_+ e_{ij}^+ + h_\times e_{ij}^\times$, where both the symmetric tensors e_{ij} are transverse and traceless. We also impose the normalization condition, $e_{ij}(\mathbf{k}) e_{ij}(-\mathbf{k})^* = 1$, for each polarization, whereas $e_{ij}^+(\mathbf{k}) e_{ij}^\times(-\mathbf{k})^* = 0$. Therefore the second order action can be written as

$$S_T = \sum_{\lambda=+, \times} \int dt d^3x a^3 \frac{M_{\text{Pl}}^2}{8} \left[\dot{h}_\lambda^2 - \frac{1}{a^2} (\partial h_\lambda)^2 \right], \quad (38)$$

so that no stability condition comes from the tensor sector.

IV. SUITABLE FORM OF POTENTIALS FOR 3-FORM INFLATION

According to the previous section, one of the no-ghost condition can be written as $V_{,y} > 0$, where $y = 6X^2 \geq 0$. The existence of ghosts in the model depends on the shape of three form potential, but not on the sign of X (as $y \propto X^2$). In fact, in order to search for the form of potentials, which makes the 3-form field ghost-free and without Laplacian instabilities ($c_X^2 \geq 0$), we need to study more in detail the evolution of y (or, equivalently, X). From the Friedmann equation, we have

$$\dot{H} = -\frac{1}{M_{\text{Pl}}^2} V_{,y} y = -\frac{1}{2M_{\text{Pl}}^2} V_{,X} X, \quad (39)$$

so that the 3-form field can play the role of a slow-rolling inflaton if $V_{,X} X / M_{\text{Pl}}^2 \ll H^2$. Substituting the above Eq. (39) into the evolution equation (14), we get

$$\ddot{X} + 3H\dot{X} + V_{\text{eff},X} = 0, \quad (40)$$

where

$$V_{\text{eff},X} = \frac{dV_{\text{eff}}}{dX} = V_{,X} \left(1 - \frac{3}{2} \frac{X^2}{M_{\text{Pl}}^2} \right), \quad (41)$$

so that the effective potential is given by

$$V_{\text{eff}}(X) = \int^X d\xi V_{,\xi} \left(1 - \frac{3}{2} \frac{\xi^2}{M_{\text{Pl}}^2} \right). \quad (42)$$

On using the dimensionless variables

$$x \equiv \frac{X}{M_{\text{Pl}}}, \quad \text{and} \quad w \equiv \frac{3x + x'}{\sqrt{6}}, \quad (43)$$

where a prime denotes a derivative with respect to $N = \ln a$, Eq. (40) can be written in the autonomous form as

$$x' = 3 \left[\sqrt{\frac{2}{3}} w - x \right], \quad (44)$$

$$w' = \frac{3}{2} \lambda(x) (1 - w^2) \left(x w - \sqrt{\frac{2}{3}} \right), \quad (45)$$

where we have introduced the function

$$\lambda \equiv \frac{V_{,x}}{V}. \quad (46)$$

In these variables the slow roll parameter can be written as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2} \lambda (1 - w^2) x. \quad (47)$$

The accelerating expansion of the universe is acquired by demanding $\epsilon \ll 1$. From this parameter, one can see that the kinetic term does not necessarily need to be small compared to the potential term, as for the standard picture of the inflaton scalar field. Conversely, it requires that $w^2 \approx 1$ when $x \lambda(x) \sim O(1)$. In order to have inflation, one needs one more requirement to guarantee that the accelerating expansion is long enough. We introduce a parameter to characterize this behavior as

$$\eta \equiv \frac{\epsilon'}{\epsilon} - 2\epsilon = (1 + c_X^2) \frac{x'}{x}, \quad (48)$$

where the inflationary period requires that $|\eta| \ll 1$. Since $c_X^2 > 0$, $|\eta|$ will be small if x'/x is small, that is x needs to be in a slow-roll regime. From Eq. (44), it implies that $|\eta| \ll 1$ will be satisfied if $x \simeq \sqrt{2/3} w \simeq \pm \sqrt{2/3}$, where we have imposed also the first slow-roll condition $|\epsilon| \ll 1$ when $x \lambda(x) \sim O(1)$.

We note that, as a consequence of the definition of w , we have $Y/(M_{\text{Pl}}H) = \sqrt{6}w$. Therefore, the Friedmann equation (10) implies

$$1 = \rho_X/(3M_{\text{Pl}}^2 H^2) = w^2 + V/(3M_{\text{Pl}}^2 H^2), \quad (49)$$

so that, if $V \geq 0$, then $0 \leq w^2 \leq 1$.

As we have already said, the field will slow-roll when it reaches the points $P \equiv (x, w) = (\pm\sqrt{2/3}, \pm 1)$ in phase-space because these points are (de Sitter) fixed points (unless $\lambda(x)$ is not finite at these points). There might be other fixed points, M , which correspond to the points where λ vanishes, that is $M \equiv (x, w) = (\bar{x}, \sqrt{3/2}\bar{x})$, where $\lambda(x = \bar{x}) = 0$. It can be seen from Eq. (41) that the points P and M are the values of X which correspond to the extrema of the effective potential.

A. Stability of the fixed points

Let us start by studying the stability of the fixed point P . By choosing $x = \pm\sqrt{2/3} + \delta x^{(1)}$, and $w = \pm 1 + \delta w^{(1)}$, we can linearize the equations of motion with respect to the small quantities δx and δw , and we find

$$d(\delta x^{(1)})/dN = \sqrt{6} \delta w^{(1)} - 3\delta x^{(1)}, \quad (50)$$

$$d(\delta w^{(1)})/dN = 0, \quad (51)$$

with solutions $\delta w^{(1)} = b_1 = \text{constant}$, $\delta x^{(1)} = \sqrt{2/3}b_1 + b_2 e^{-3N}$, and $b_{1,2}$ are (small) initial conditions. The fact that $d^2(\delta w^{(1)})/dN^2 = 0$, implies that, at linear order, we cannot deduce whether the fixed point is stable or not. In order to check the stability of this fixed point, one needs to study also the second-order solution. Therefore, by re-iterating the method, and choosing $x = \pm\sqrt{2/3} + \delta x^{(1)} + \delta x^{(2)}$, and $w = \pm 1 + \delta w^{(1)} + \delta w^{(2)}$, we find

$$d(\delta x^{(2)})/dN = \sqrt{6}\delta w^{(2)} - 3\delta x^{(2)}, \quad (52)$$

$$d(\delta w^{(2)})/dN = -\lambda(\pm\sqrt{2/3})[3b_1b_2e^{-3N} + 2\sqrt{6}b_1^2], \quad (53)$$

which can be solved as

$$\delta w^{(2)} = b_3 + \lambda(\pm\sqrt{2/3})b_1(b_2e^{-3N} - 2\sqrt{6}b_1N), \quad (54)$$

$$\delta x^{(2)} = \sqrt{2/3}b_3 + \lambda(\pm\sqrt{2/3})b_1^2(4/3 - 4N) + \sqrt{6}\lambda(\pm\sqrt{2/3})b_1b_2Ne^{-3N} + b_4e^{-3N}, \quad (55)$$

where $b_{3,4} = \mathcal{O}(b_1^2)$. These second order solutions show that, at second order, the fixed point is unstable (due to the term proportional to N). The fact that this instability appears at second order means that the instability will in general evolve slowly. This slow instability will make inflation end eventually.

As for the fixed point $M = (x, w) = (\bar{x}, \sqrt{3/2}\bar{x})$, where $\lambda(\bar{x}) = 0$, then by choosing $x = \bar{x} + \delta x$, and $w = \sqrt{3/2}\bar{x} + \delta w$, we find the linearized equations

$$\delta x' = \sqrt{6}\delta w - 3\delta x, \quad (56)$$

$$\delta w' = -\frac{\sqrt{6}}{8}(2 - 3\bar{x}^2)^2\bar{\Gamma}\delta x, \quad (57)$$

where

$$\bar{\Gamma} = \left. \frac{V_{,xx}}{V} - \left(\frac{V_{,x}}{V} \right)^2 \right|_{x=\bar{x}} = \left. \frac{V_{,xx}}{V} \right|_{x=\bar{x}}. \quad (58)$$

The solution leads to

$$\delta x = d_1 e^{-N(3+\gamma)/2} + d_2 e^{-N(3-\gamma)/2}, \quad (59)$$

where

$$\gamma = \sqrt{9 - 3\bar{\Gamma}(3\bar{x}^2 - 2)^2}. \quad (60)$$

An instability will appear if $\gamma > 3$, or $\bar{\Gamma} < 0$.

B. Mexican-hat potential

Let us start with the Mexican-hat type potential of the form

$$V = V_0(x^2 - c^2)^2. \quad (61)$$

This potential yields

$$V_{,y} = \frac{V_{,X}}{12X} = \frac{V_{,x}}{12M_{\text{Pl}}^2x} = \frac{V_0}{3M_{\text{Pl}}^2}(x^2 - c^2), \quad \text{and} \quad c_X^2 = \frac{3x^2 - c^2}{x^2 - c^2}. \quad (62)$$

Hence, the ghost will not exist if $|x| > |c|$ and for this range of x the 3-form is stable, i.e., $c_X^2 > 0$. For this form of the potential, the effective potential has 5 extremum points at $x = 0$, $x = \pm|c|$ and $x = \pm\sqrt{2/3}$. If $|c| < \sqrt{2/3}$, the points $x = \pm|c|$ are the minimum while $x = \pm\sqrt{2/3}$ are the maximum points of the potential. The points $x = \pm|c|$ and $x = \pm\sqrt{2/3}$ become the maximum and minimum points respectively when $|c| > \sqrt{2/3}$. In Figure (1) the potentials for the case $|c| < \sqrt{2/3}$ and $|c| > \sqrt{2/3}$ are plotted. In the case where $|c| < \sqrt{2/3}$, the field x that starts at points A and C usually evolves to point B. Roughly speaking, the field can climb up the effective potential from the point C to A because x' is negative (when the field is on the right side of the point A), and $|w|$ is never larger than unity (as V in this range is always positive).

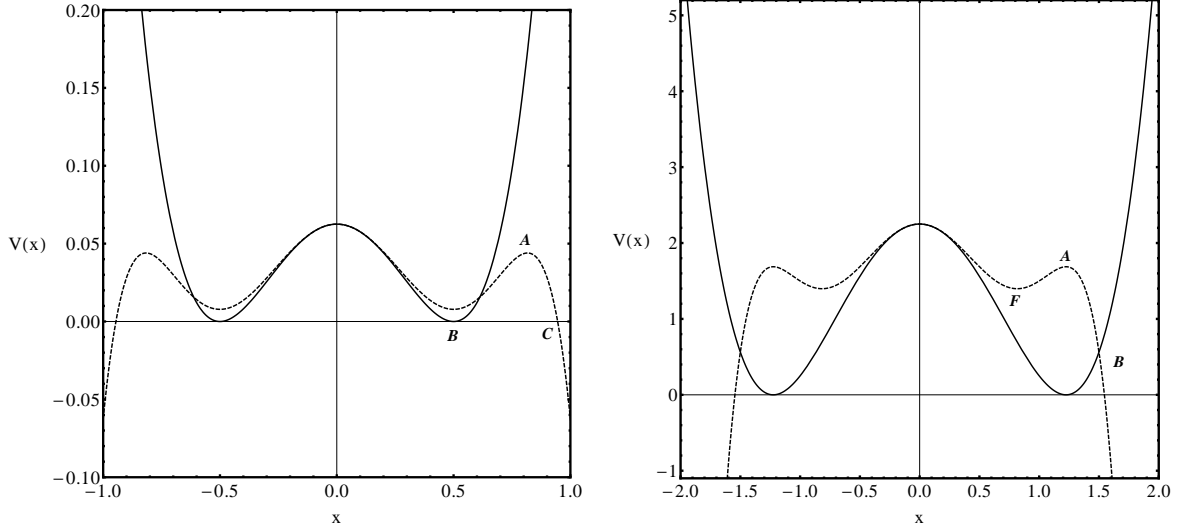


Figure 1. The potential $V = V_0(x^2 - c^2)^2$ is represented by solid line while the effective potential is represented by dashed line. In the left panel $c = 1/2 < \sqrt{2/3}$ while $c = \sqrt{3/2} > \sqrt{2/3}$ in the right panel.

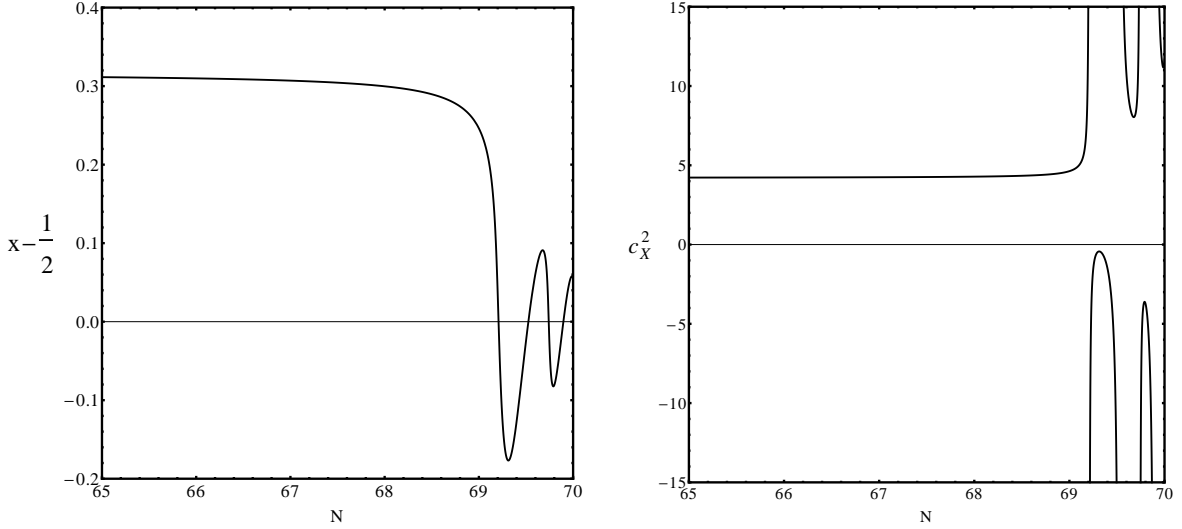


Figure 2. The evolution of x for the potential $V = V_0(x^2 - c^2)^2$, and $c = 1/2$. We chose initial conditions $x(0) \approx 0.816188$, and $w(0) \approx 0.99962$, so that $\epsilon(0) \approx 9.289 \times 10^{-4}$. We notice that as x becomes less than one half, then a ghost appears, and, at the same time, c_X^2 diverges and changes sign.

By using the previous qualitative analysis, we can generalize this behavior to other potential forms. Since $\epsilon \propto \lambda(x) x (1 - w^2)$, the field can drive inflation if $|w| \sim 1$ and $\lambda(x) x$ is not large. From equation (48), the inflation will be long enough if $|x| \gtrsim \sqrt{2/3}$ initially. The autonomous equations imply that $x' < 0$ if $x > \sqrt{2/3}$ and $x' > 0$ if $x < -\sqrt{2/3}$. When we initially put the field in the region $|x| > \sqrt{2/3}$, the field will evolve to the de Sitter point P , where the condition $|\eta| \ll 1$ is satisfied. However, since P is not a stable fixed-point, the field will evolve by crossing P , and entering the region $|x| < \sqrt{2/3}$. If there is a minimum in the region $|x| < \sqrt{2/3}$, the field will eventually oscillate around this minimum, so that a mechanism to end the inflation is allowed. We note that condition $|\eta| \ll 1$ can be satisfied even if we initially put the field in the region $x \lesssim \sqrt{2/3}$. However, we may need to fine tune the initial value of w close to unity.

For this potential form, when the field reaches the point B it will oscillate around this point. Nevertheless, the field becomes ghost when it moves to the left of the point B, so that the field cannot oscillate around the minimum of the effective potential without becoming ghost.

In Figure 2 we plot the evolution of x for $c = 1/2$ and some initial conditions compatible with slow-roll, to explicitly

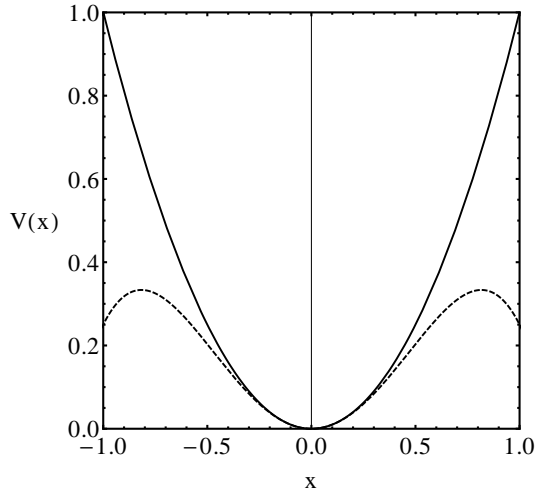


Figure 3. The potential $V(x) \propto x^2$ is shown here. In the figure, the bare potential and the effective potential are represented by a solid and dashed line respectively.

show, also numerically, that the ghost (and the Laplacian instabilities) will, in general, appear.

For the case $|c| > \sqrt{2/3}$, the right panel of Figure (1) shows that the ghost will not exist and $c_X^2 > 0$ if the field x evolves between A and B. Nevertheless, the point F is the stable fixed point in this case, so that the field that starts from point B will evolve towards point F. This implies that, also in this case, the field will become a ghost, eventually.

C. Power-law potential

We would like to present here a class of potentials which can give a stable inflation scenario. On choosing a power law potential of the form $V \propto y^p$, we immediately notice that $Q \propto y^{p-1}$, which in general vanishes (for $p > 1$) or diverges (for $p < 1$) as $y \rightarrow 0$, unless $p = 1$. Since we will focus on values $p \geq 1$, most of the potential will allow the field to cross this value ($y = 0$), so that Q will vanish in the origin. This property represents a problem, in general, as this means that, at that point, the second order Lagrangian vanishes (as c_X^2 remains finite for $V \propto y^p$), and the theory becomes strongly coupled, i.e. higher order corrections become dominant. In order to avoid this possible issue we propose the following generalized power law potential¹,

$$V(x^2) = V_0 [(x^2)^p + b x^2], \quad (63)$$

where p is a constant which can be, as for now, positive or negative, whereas $b > 0$. For this form of the potential, we have that the no-ghost condition

$$\frac{V_{,x}}{x} = 2V_0[p(x^2)^{p-1} + b] > 0, \quad (64)$$

is always positive for $p \geq 0$, and also finite for $p \geq 1$. For this reason, from now on, we will consider only the case $p \geq 1$. On the other hand, since

$$c_X^2 = \frac{(2p-1)p(x^2)^{p-1} + b}{p(x^2)^{p-1} + b}, \quad (65)$$

c_X^2 will be always positive and finite for $p \geq 1$. The bottom line is that for $p \geq 1$ the model is free from instabilities for any real value of x , that is for any dynamics. It should be noticed that for the value $p = 1$, the potential reduces to a quadratic power law potential. On the de Sitter fixed point P , we have

$$c_X^2(x = \pm\sqrt{2/3}) = \frac{(2p-1)p(2/3)^{p-1} + b}{p(2/3)^{p-1} + b} \geq 1, \quad (66)$$

¹ In general, we can choose a larger class of potentials given as $V(y) = V_0 (c y + \sum_i c_i y^{p_i})$, where $c > 0$, $c_i \geq 0$, and $p_i \geq 1$.

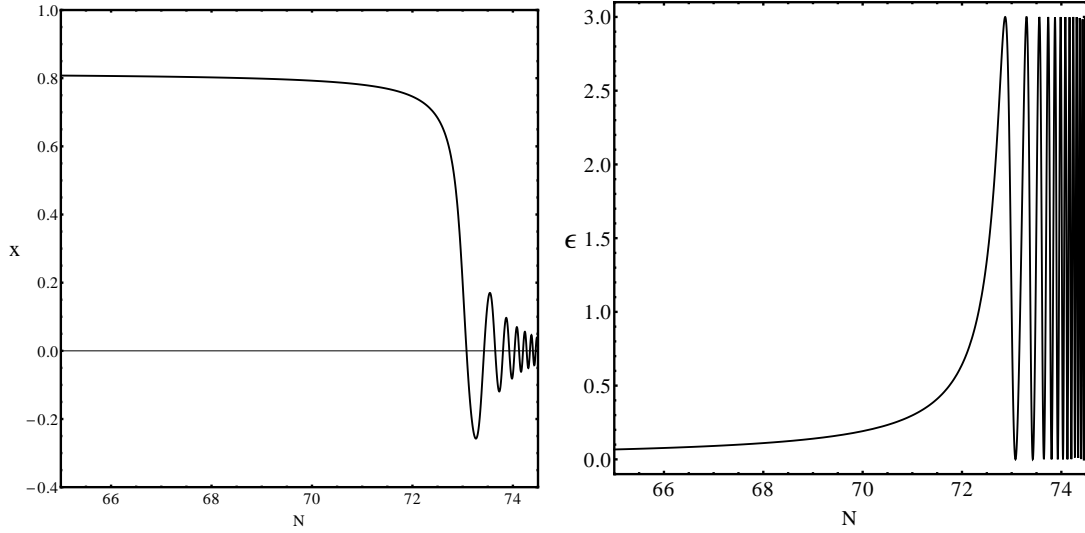


Figure 4. The evolution of x and ϵ for the potential $V = V_0(1+b)x^2$, and $p = 1$. We chose initial conditions $w(0) = 0.99$, and $x(0) = 3\sqrt{2/3}w(0)$. When the field x start to oscillate around the minimum, the parameter ϵ will start to oscillate around $\epsilon = 3/2$. Thus the inflation will end when $\epsilon \sim 1$ corresponding to $N \sim 72$.

and the inequality holds for $p \geq 1$, and $b > 0$.

Because of Eq. (63), it can be shown that

$$V_{\text{eff},x} = 2V_0 \left(1 - \frac{3}{2}x^2\right) [px^{2p-1} + bx], \quad (67)$$

and

$$V_{\text{eff}} = V_0 \left[\frac{x^{2p}}{2(p+1)} [2 + p(2 - 3x^2)] + bx^2 \left(1 - \frac{3}{4}x^2\right) \right], \quad (68)$$

so that the extremum points of this potential occur at $x = \pm\sqrt{2/3}$, and $x = 0$. We also notice that

$$\lambda(x)x = \frac{2[p(x^2)^{p-1} + b]}{[(x^2)^{p-1} + b]}, \quad (69)$$

which, for $p \geq 1$, is positive and finite for all x . Furthermore $\lim_{x \rightarrow 0} \lambda x = 2$.

For illustration, we consider here the simplest case $p = 1$ whereas, in the next section, we will describe in more detail the case $p = 2$ (quartic potential). The potential and effective potential for the $p = 2$ case are plotted in Figure 3. Similar to the case of Mexican-hat type potential, the field that starts at $|x| > \sqrt{2/3}$ with $|w| \sim 1$ is able to drive long-enough inflation. However, this time, the field rolls down the potential from the points A or B and then oscillates around the minimum of the potential without ghosts ($V_{,yy} = V_0(1+b)/6 > 0$) or Laplacian instabilities ($c_X^2 = 1$). We also use a direct numerical integration to confirm both the slow roll and the oscillatory regimes as shown in Figure 4. From the evolution of ϵ in the right panel, inflation ends at $N \sim 72$ corresponding to $\epsilon \sim 1$.

D. Quartic potential

We study here a particular case of the power law potential introduced in the previous section, namely

$$V = V_0(x^4 + bx^2), \quad (70)$$

and we plot it (together with its effective potential) in Figure 5.

In this case, we also have

$$\frac{3M_{\text{Pl}}^2 H^2}{V_0} = \frac{x^4 + bx^2}{1 - w^2}, \quad (71)$$

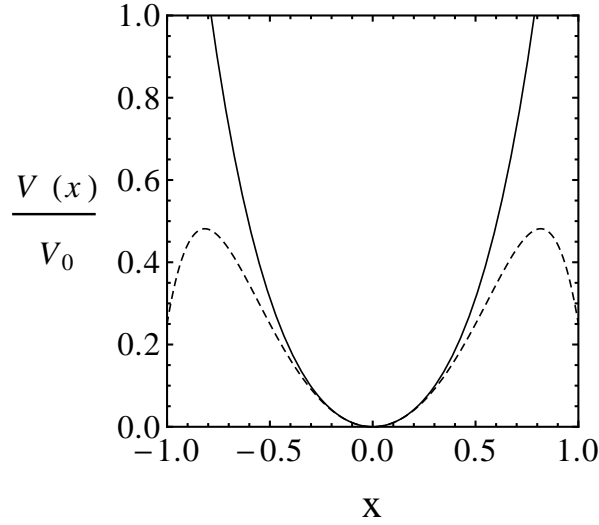


Figure 5. The potential $V = V_0(x^4 + bx^2)$, with $b = 1$ (the continuous and black curve), together with the effective potential V_{eff} (represented by the dashed curve).

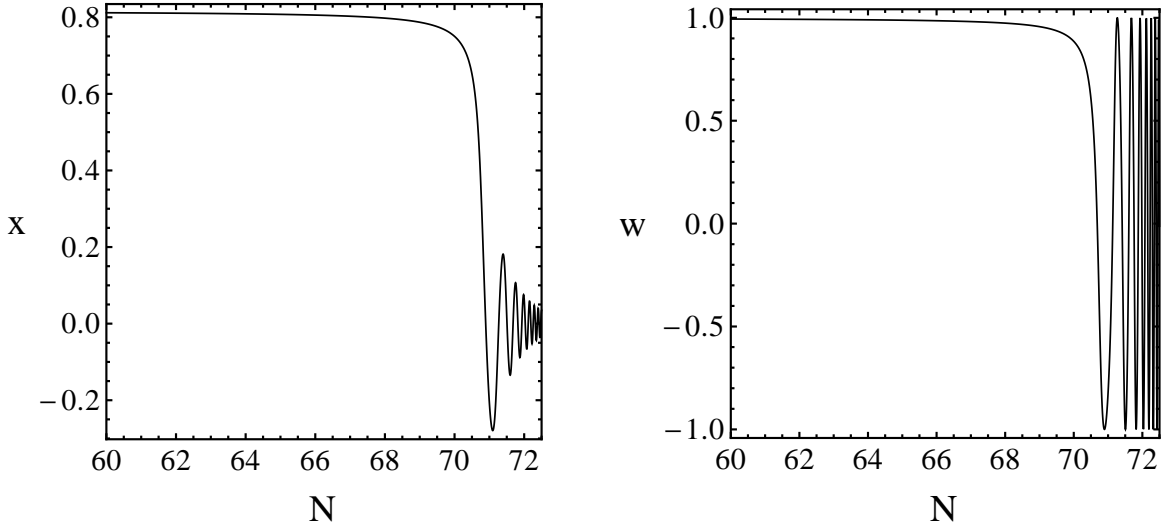


Figure 6. The evolution of x and w for the potential $V = V_0(x^4 + bx^2)$, and $b = 1$. We chose initial conditions $w(0) \approx 0.9991$, and $x(0) \approx 0.8158$.

so that, by introducing a dimensionless cosmic time t , we can write

$$\frac{dx}{dt} = \frac{M_{\text{Pl}} H}{\sqrt{V_0/3}} \frac{dx}{dN} = \sqrt{\frac{x^4 + bx^2}{1 - w^2}} \frac{dx}{dN}. \quad (72)$$

In Figure 6, we show the evolution for both x , and w . The field slow-rolls until an oscillatory regime starts, making inflation end.

During the slow-roll regime the propagation speed takes the value $c_X^2 \approx \frac{3(4+b)}{4+3b}$, whereas, as the solution starts oscillating, $c_X^2 \rightarrow 1$. This behavior is confirmed in Figure 7.

Finally, we show in Figure 8 that after inflation ends, there is an oscillatory regime which mimics a dust dominated universe as we have $H^2 \propto a^{-3} \propto e^{-3N}$. This behavior is similar to the standard single-field inflationary models, and this is not surprising, because as $x \rightarrow 0$, we find $V_{\text{eff}} \approx bx^2$; so that the equation of motion for the field, Eq. (14), reduces to

$$\ddot{x} + 3H\dot{x} \approx -bx, \quad \text{for } |x| \ll 1, \quad (73)$$

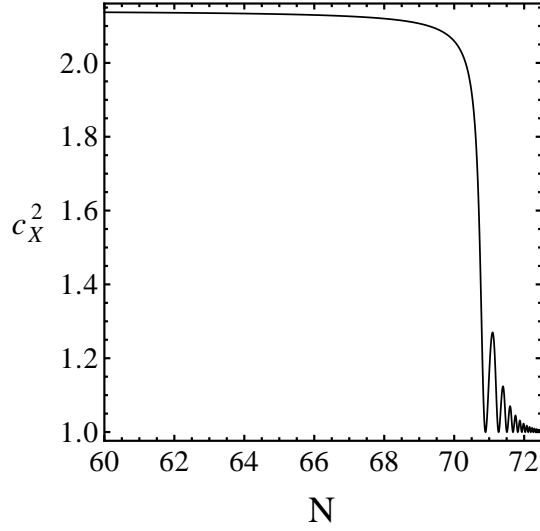


Figure 7. The evolution of c_X^2 for the potential $V = V_0(x^4 + bx^2)$, and $b = 1$. The model does not possess any instability.

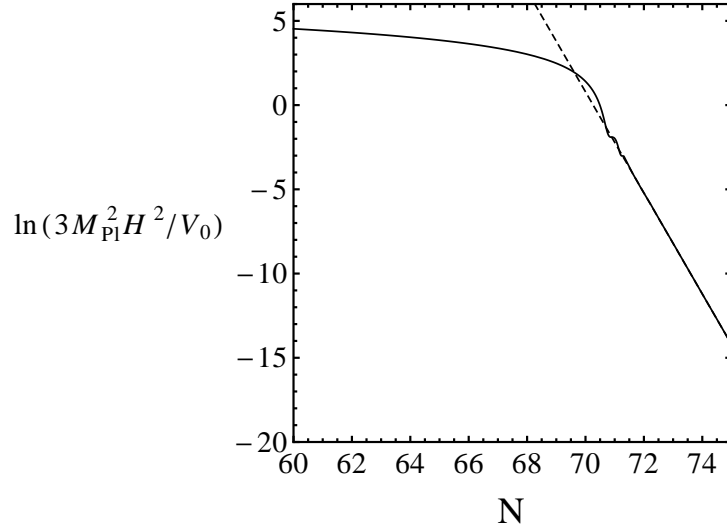


Figure 8. The evolution of $3M_{\text{Pl}}^2 H^2 / V_0$ for the potential $V = V_0(x^4 + bx^2)$, and $b = 1$ (continuous black curve). This figure shows that after inflation ends (around $N \simeq 70$), the universe enters a matter dominated epoch, as the curve approaches a dashed line, which represents the line $\ln(3M_{\text{Pl}}^2 H^2 / V_0) = -3N + \text{constant}$. This means that after inflation $H^2 \propto e^{-3N} \propto a^{-3}$.

which exactly matches the equation of motion for standard inflation in the presence of a quadratic inflaton potential. In other words, the dynamics of the 3-form, for $x \rightarrow 0$, tends to be more and more identical to the dynamics of a single scalar field oscillating around the minimum of a quadratic potential.

After inflation ends, during the oscillatory regime, in Figure 6, we see that $x \rightarrow 0$, whereas w oscillates between -1 and 1 . Furthermore, we also find that $dx/dt \rightarrow 0$ together with x , whereas dw/dt keeps oscillating, remaining finite, as shown in Figure 9.

E. Gaussian potential

We now consider the exponential potential

$$V = V_0(e^{\nu y/6} - 1) = V_0(e^{\nu x^2} - 1), \quad (74)$$

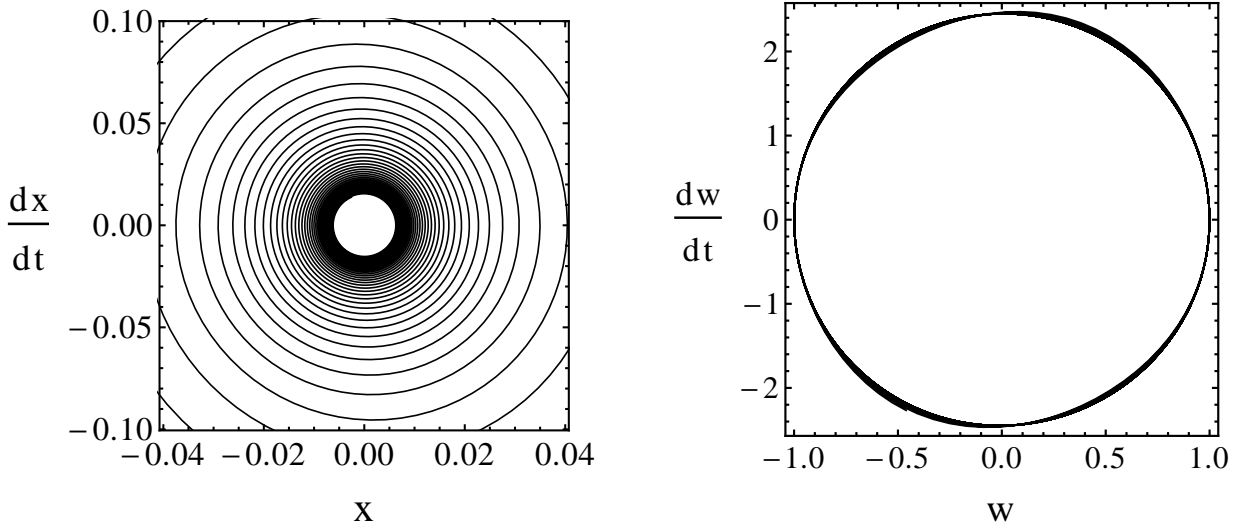


Figure 9. The evolution of dx/dt as a function of x , and dw/dt as a function of w for the potential $V = V_0(x^4 + bx^2)$, and $b = 1$ during the oscillatory regime. We chose initial conditions so that at $t = 1$, the values of x and w correspond respectively to $x(N = 72) \approx 0.0644$, and $w(N = 72) \approx -0.46$ of Figure 6. We stop the integration at $t = 100$. This figure shows that x spirals, whereas w keeps on oscillating during the matter dominated regime.

where ν is a constant parameter which can be positive or negative. For this form of potential, we have

$$\frac{V_{,x}}{x} = 2\nu V_0 e^{\nu x^2}, \quad (75)$$

so that the ghost will not exist if ν is positive. The speed of propagation in this case is given by

$$c_X^2 = 1 + 2\nu x^2. \quad (76)$$

This implies that if the ghost does not exist, c_X^2 is always positive. Substituting Eq. (74) into Eq. (41), one gets

$$V_{\text{eff},x} = 2\nu x V_0 e^{\nu x^2} \left(1 - \frac{3}{2} x^2\right), \quad (77)$$

or

$$V_{\text{eff}} = -\frac{V_0}{2\nu} \left\{3 + 2\nu + e^{\nu x^2} [(3x^2 - 2)\nu - 3]\right\}, \quad (78)$$

and we plot it, together with the bare potential, in Figure 10. We also notice that as $x \rightarrow 0$, then $V_{\text{eff}} \simeq V_0 \nu x^2$, so that we expect an oscillatory regime to take place, ending inflation.

It is easy to see that the effective potential has the extremum at $x = \pm\sqrt{2/3}$ and $x = 0$. Similar to the analysis for the previous potentials, the field can drive inflation when we initially put it in the region satisfying the condition $\epsilon \ll 1$, e.g. $|x| \gtrsim \sqrt{2/3}$ and $|w| \sim 1$. The condition $|\eta| \ll 1$ will be satisfied when the field is frozen nearly $x = \pm\sqrt{2/3}$. Since $x = \pm\sqrt{2/3}$ are not stable fixed points, the field can continuously evolve through $x = \pm\sqrt{2/3}$ and then oscillates about $x = 0$ eventually. This behavior is also shown by using numerical integration methods as seen in Figure 11. Because of this behavior, the speed of propagation will be approximately equal to $c_X^2 \approx 1 + 4\nu/3$ in the slow-roll regime, whereas $c_X^2 \rightarrow 1$, as $x \rightarrow 0$. In Figure 12, we also show the behavior of the Hubble parameter during the oscillatory regime, confirming that a matter-dominated era takes place during this epoch.

Finally, we show the trajectory of dx/dt and x , together with dw/dt and w in Figure 13.

V. GENERAL CONSIDERATIONS AND CONCLUSIONS

From the investigation of the previous section, one can see that the viable 3-form models can be characterized by the shape of their potential. The study of power-law potentials suggests that the viable potential form which is free

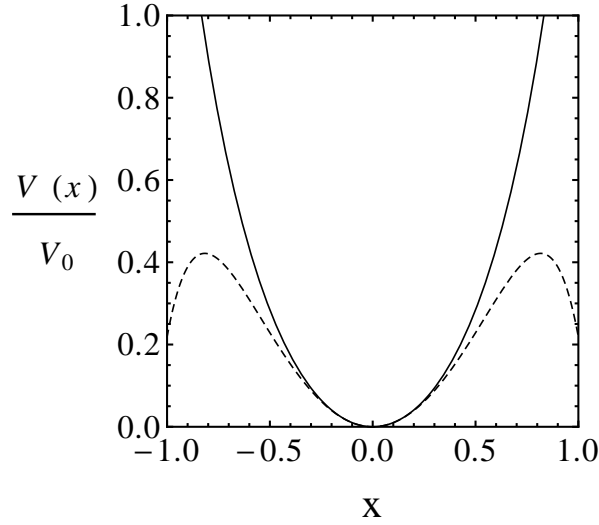


Figure 10. The potential $V(x) = V_0(e^{\nu x^2} - 1)$ is represented by a solid line (for $\nu = 1$), whereas the effective potential is represented by a dashed line.

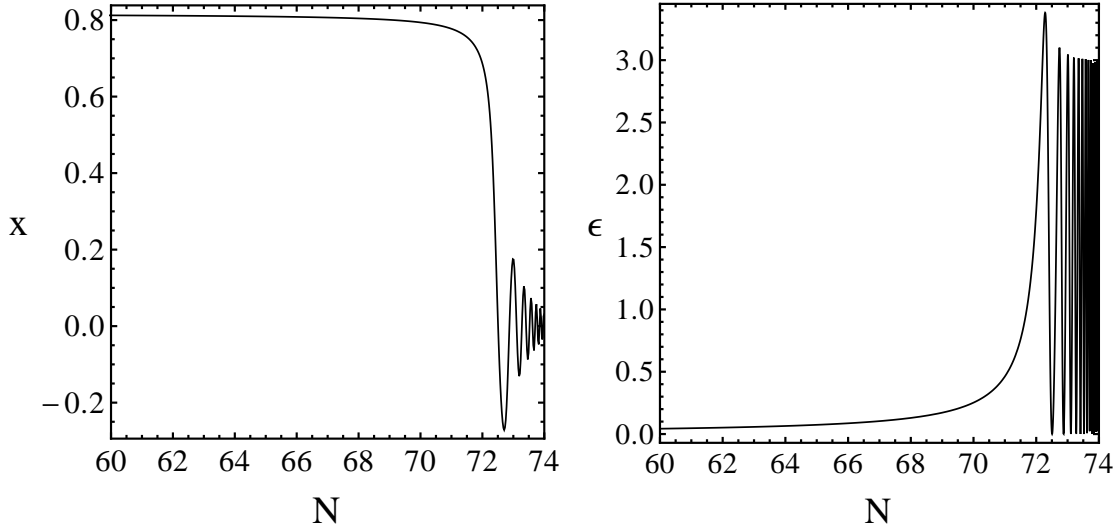


Figure 11. The evolution for x (left panel) and ϵ (right panel) for the potential $V_0(e^{\nu x^2} - 1)$, and $\nu = 1$. We chose initial conditions $w(0) \approx 0.999$, and $x(0) \approx 0.815$.

from ghosts and Laplacian instability should have the local minimum at $x = 0$. This is because when $V_{,x}$ changes sign around the minimum point (as V is, by construction, an even function of x , $V(x) = V(-x)$), x also changes sign such that $V_{,x}/x = 12V_{,y}$ is always positive (where $y = 6X^2$, and $x = X/M_{\text{Pl}}$). In this situation, $c_X^2 > 0$ around $x = 0$ because $V_{,xx} > 0$. The speed of propagation is still positive as long as x remains significantly far from the nearest local maximum (if it exists) of the potential along the trajectory of motion. Hence, if the bare potential has no local maxima between $x = \pm\sqrt{2/3}$, the field can evolve between $x = \pm\sqrt{2/3}$ (which, in turn, are the local maxima of the effective potential) without giving rise to ghosts or Laplacian instabilities.

In section IV we have proposed a class of potentials which are free of instabilities, can drive inflation, and provide a final stage of matter-dominated-like oscillatory epoch, during which reheating can occur. A simple example for such a potential is

$$V = V_0 (bx^2 + (x^2)^p), \quad \text{with} \quad b > 0, \quad p \geq 1. \quad (79)$$

We have introduced this form for the potential because, for simple power-law monomials, i.e. $V \propto y^p$, with $p > 1$, the second order action for the perturbations given in Eq. (26) will vanish at $y = 0$ since $Q \propto V_{,y} = 0$. This corresponds,

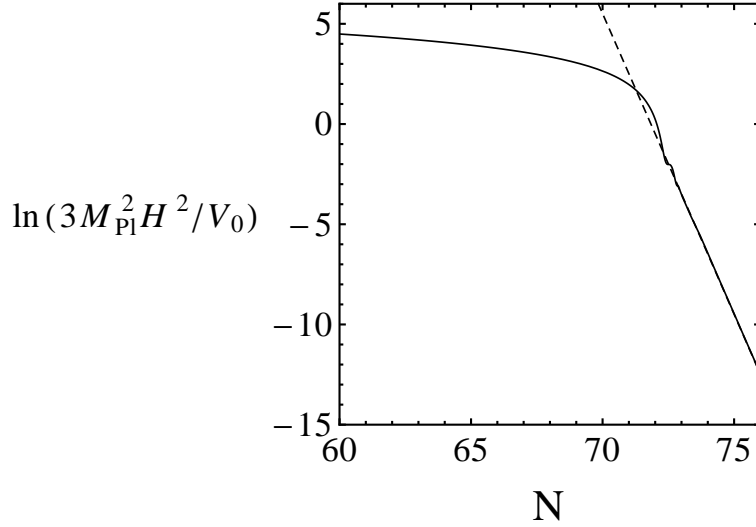


Figure 12. The evolution of $3M_{\text{Pl}}^2 H^2 / V_0$ for the potential $V = V_0(e^{\nu x^2} - 1)$, and $\nu = 1$ (continuous black curve). This figure shows that after inflation ends (around $N \simeq 72$), the universe enters a matter dominated epoch, as the curve approaches a dashed line, which represents the line $\ln(3M_{\text{Pl}}^2 H^2 / V_0) = -3N + \text{constant}$. This means that after inflation $H^2 \propto e^{-3N} \propto a^{-3}$.

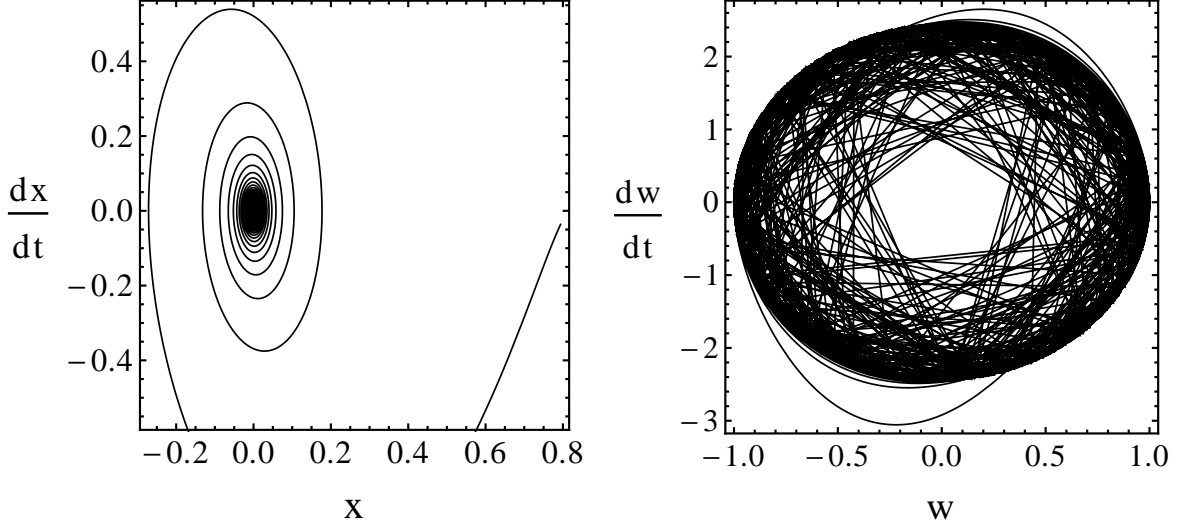


Figure 13. Phase space plot for the variables dx/dt and x (left panel), and for dw/dt and w (right panel), during the oscillatory epoch.

in general, to a strong coupling limit for the theory. One can avoid this situation by modifying the power-law potential as in Eq. (79). There is no fixed-point at $x = 0$ for this form of the potential. Therefore, the field can oscillate around $x = 0$ to provide the mechanism to end the inflation without reaching $Q = 0$ at $y = 0$. More in detail, according to the previous section, the points in region $|x| > \sqrt{2/3}$ (unstable slow-roll fixed point of the dynamical equations of motion) will be forced to move to the region $|x| < \sqrt{2/3}$, and the inflationary period will be long enough if the field x starts at $|x| = x_s > \sqrt{2/3}$ with $|w| \sim 1$, where $w \propto x' + 3x$. The bottom line is that, in general, the 3-form field can drive long enough inflation without the ghosts or instabilities if its potential has local minimum at $x = 0$ and has no local maximum between $x = \pm x_s$.

We also give another working example, the Gaussian potential, here defined as

$$V = V_0(e^{\nu x^2} - 1), \quad (80)$$

which has similar properties to the power-law case discussed above. In fact, a long-enough slow-roll regime is followed by an oscillatory epoch where inflation ends.

Even if avoiding ghosts ($Q > 0$) and Laplacian instabilities ($c_X^2 < 0$) are necessary conditions to be satisfied, they are not, however, sufficient, in general, to have a successful period of inflation. In other words, it is not assured that inflation ends for other classes of potentials which are, on the other end, free from instabilities.

If the potential $V(y)$ is such that for $y \geq 0$, it satisfies the conditions $V \geq 0$, $V_{,y} > 0$, and $V_{,yy} \geq 0$, then no instabilities arise, as already said. However, if we also impose that as $y \rightarrow 0$, we have $V(y) \simeq cy$, where c is a positive constant, then for $x \approx 0$ (and this point can be reached), $V_{\text{eff}} \propto x^2$, so that in general, an oscillatory epoch can take place, ending inflation.

We have investigated the stability of the perturbations for a minimally coupled 3-form, whose action has a standard kinetic term and a generic potential function. We have found the conditions for which the inflationary dynamics can be stable, and gave some classes of potentials which can provide enough inflation without generating ghosts or Laplacian instabilities. We will leave the question to constrain the parameter space for this potentials by using the bounds on the spectral index and tensor-to-scalar ratio to a future research project.

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Appendix A: The dual theory

It is possible to define the 1-form or vector dual to the 3-form as

$$A_{\alpha\beta\gamma} = E_{\alpha\beta\gamma\delta} B^\delta = \sqrt{-g} \epsilon_{\alpha\beta\gamma\delta} B^\delta, \quad (\text{A1})$$

where $E_{\alpha\beta\gamma\delta}$ is the Levi-Civita antisymmetric tensor on curved backgrounds, which on Minkowski reduces to $\epsilon_{\alpha\beta\gamma\delta}$ (with $\epsilon_{0123} = 1 = -\epsilon^{0123}$). Then we also have $E^{\alpha\beta\gamma\delta} = \epsilon^{\alpha\beta\gamma\delta} / \sqrt{-g}$. It is easy to show that $\nabla_\mu E_{\alpha\beta\gamma\delta} = 0$. In the following we will make use of the following relations $\epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\beta\gamma\delta} = -6\delta_\mu^\alpha$, and $\epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\gamma\delta} = -2(\delta_\mu^\alpha \delta_\nu^\beta - \delta_\nu^\alpha \delta_\mu^\beta)$. Therefore we obtain

$$A_{\alpha\beta\gamma} A^{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma\delta} \epsilon^{\alpha\beta\gamma\mu} B_\mu B^\delta = -6 B_\mu B^\mu. \quad (\text{A2})$$

We also have

$$\begin{aligned} -\frac{1}{48} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} &= -\frac{1}{2} F_{0123} F^{0123} = -\frac{1}{2} (\epsilon_{1230} \nabla_0 B^0 - \epsilon_{0123} \nabla_3 B^3 + \epsilon_{3012} \nabla_2 B^2 - \epsilon_{2301} \nabla_1 B^1) \\ &= \times (\epsilon^{1230} \nabla^0 B_0 - \epsilon^{0123} \nabla^1 B_1 + \epsilon^{3012} \nabla^2 B_2 - \epsilon^{2301} \nabla^3 B_3) = \frac{1}{2} (\nabla^\mu B_\mu)^2, \end{aligned} \quad (\text{A3})$$

so that the action is equivalent to the following one

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + \frac{1}{2} (\nabla^\mu B_\mu)^2 - V(B_\mu^2) \right], \quad (\text{A4})$$

which shows that the 3-form action is classically equivalent to a particular class of vector-tensor theories. Relation (A1) can be inverted to give

$$B^\mu = \frac{1}{3!} \frac{1}{\sqrt{-g}} \epsilon^{\mu\alpha\beta\gamma} A_{\alpha\beta\gamma}, \quad (\text{A5})$$

therefore, once the tensor \mathbf{A} is known we can uniquely find \mathbf{B} . At the level of the perturbations we find

$$\delta B^\mu = \frac{1}{3!} \frac{\epsilon^{\mu\alpha\beta\gamma}}{\sqrt{-g}} \left[\frac{A_{\alpha\beta\gamma}}{2} g_{\rho\sigma} \delta g^{\rho\sigma} + \delta A_{\alpha\beta\gamma} \right], \quad (\text{A6})$$

which is valid on any background.

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